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INTEGRAL SPECTRUM ABSORPTION COEFFICIENTS OF A MEDIUM

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Abstract—The absorption of a medium in a spectral band, in a total spectrum, and emissivity are described by converging series. The series coefficients are studied. The sets of approximate formulae are constructed by convolution of residues of series. These formulae are verified for emissivity of water vapour. Two lower order coefficients known in the literature are discussed as a particular case.

NOMENCLATURE

A_j,	integral absorption of a band [cm^{-1}];
$\bar{A}_j$,	$A_j/\Delta\omega$;
E_1,	$\int_1^\infty t^{-1} \exp(ut)\,dt$;
L,	Ladenburg–Reiche's function;
I_ω,	spectral intensity [cmW/m^2 ster];
$I_{0\omega}$, I_{0j},	Planck function for a wave number ω, for a band centre j [cm W/m^2 ster];
S_j,	integral intensity of a band j [cm^{-1}/m. atm];
S_{0j},	integral intensity of a band j [cm^{-1}/gm^{-2}];
T, t,	medium temperature [°K, °C];
T_0,	temperature of a black emitter [°K];
a_0,	gas absorptivity for a black emitter [dimensionless];
a,	gas absorptivity for arbitrary spectrum flow;
p,	partial pressure [atm];
x,	reduced beam path with respect to a partial pressure, $\int p\,dl$ [m.atm];
y,	$v/\Delta\omega_j$;
y',	$v/\Delta\omega'_j$;
α_{ij},	coefficients for radiation, absorption and reradiation in a band j in the case of multiple interactions between a flow and substance, if $i > 1$ [(m. atm)$^{-1}$];
$\alpha_{1j} \equiv S_j$,	
α_i, α_i^*,	coefficients similar in their meaning to α_{ij} but for a total spectrum [(m. atm)$^{-1}$];
$\alpha_c \equiv \alpha_1$,	radiation coefficient or mean absorption Planck coefficient;
$\alpha_* \equiv \alpha_2$,	initial absorption coefficient for a flow with elementary volume spectrum at the same temperature;
α_ω,	spectral absorption coefficient [(m. atm)$^{-1}$];
γ	$= 1{\cdot}4388$;
σ,	$= 5{\cdot}68 \cdot 10^{-8}$ W/m^2 grad^4;
$\varepsilon(x, T)$,	emissivity of section x over a total spectrum;
μ_0,	kilomole mass [kg/kmole];
v,	$\lvert\omega - \omega_0\rvert$;
ω,	wave number [cm^{-1}];
ω_0;	position of a band edge;
$\Delta\omega_j$,	band width parameter of a reconstructed band [cm^{-1}];
$\Delta\omega'_j$,	total width of a re-constructed band.

1. INTRODUCTION

IN PRACTICAL calculations of heat transfer by radiation the mode of reducing a nonhomogeneous beam path in a medium to a homogeneous one is widely used. The extreme simplification consists in introducing an effective volume temperature. In these cases the calculation is based on emissivity, absorptivities of a gas for a black and non-black radiation (ε, a_0 and a)

$$\varepsilon = \frac{\pi}{\sigma T^4}\int\limits_0^\infty I_{0\omega}A_\omega\,\mathrm{d}\omega;\quad A_\omega = 1 - \exp(-\alpha_\omega x)$$

$$a_0 = \frac{\pi}{\sigma T_0^4}\int\limits_0^\infty I_{0\omega}(T_0)\,A_\omega(x, T)\,\mathrm{d}\omega;$$

$$a = \frac{1}{I}\int\limits_0^\infty I_\omega A_\omega(x, T)\,\mathrm{d}\omega.$$

For a band spectrum

$$\varepsilon = \frac{\pi}{\sigma T^4}\sum_j I_{0j}A_j.$$

The quantities a_0 and a may be written similarly. The integral absorption in a band is of fundamental significance

$$A_j = \int\limits_{\Delta\omega' j}[1 - \exp(-\alpha_\omega x)]\,\mathrm{d}\omega.$$

This paper gives a general procedure of approximate description of the functions ε, a_0, a, A_j. The use of the integral absorption coefficients with respect to the spectrum is peculiar of this procedure. The properties of the coefficients are studied, and lower-order coefficients are evaluated. Numerical calculations for carbon dioxide and water vapour are made.

2. FORMAL RELATIONS

All the characteristics under consideration are written in the form of power series on the basis of the expansion

$$A_\omega = \sum_{i=1}^{\infty}(-1)^{i+1}\alpha_\omega^i x^i/i!.$$

For further transformations it is useful to introduce the numbers m_i

$$A_\omega = \sum_{i=1}^{\infty}(-1)^{i+1}m_i\alpha_\omega^i x^i/i!m_i, \qquad m_1 = 1.$$

The absorption coefficients integral over the absorption spectrum are determined by the relation

$$\prod_{k=1}^{i}\alpha_k = \frac{m_i}{i!}\frac{\pi}{\sigma T^4}\int\limits_0^\infty I_{0\omega}\alpha_\omega^i\,\mathrm{d}\omega.$$

Their use leads to the series

$$\varepsilon = \alpha_1 x - \alpha_1\alpha_2 x^2/m_2 + \ldots(-1)^{i+1} \times \frac{x^i}{m_i}\prod_{k=1}^{i}\alpha_k\ldots \qquad (1)$$

For the band spectrum

$$\prod_{k=1}^{i}\alpha_i = \frac{m_i}{i!}\frac{\pi}{\sigma T^4}\sum_j I_{0j}\int\limits_{\Delta\omega' j}\alpha_\omega^i\,\mathrm{d}\omega. \qquad (2)$$

The quantities a_0, a, A_j may be written similarly

$$a_0 = \alpha_1^* x - \alpha_1^*\alpha_2^* x^2/m_2 + \ldots(-1)^{i+1} \times \frac{x^i}{m_i}\prod_{k=1}^{i}\alpha_k^*\ldots$$

$$\prod_{k=1}^{i}\alpha_k^* = \frac{m_i}{i!}\frac{\pi}{\sigma T_0^4}\int\limits_0^\infty I_{0\omega}(T_0)\,\alpha_\omega^i(T)\,\mathrm{d}\omega$$

$$A_j = \alpha_{1j}x - \alpha_{1j}\alpha_{2j}x^2/m_2 + \ldots(-1)^{i+1} \times \frac{x^i}{m_i}\prod_{k=1}^{i}\alpha_{kj}\ldots \qquad (3)$$

where

$$\prod_{k=1}^{i}\alpha_{kj} = \frac{m_i}{i!}\int\limits_{\Delta\omega' j}\alpha_\omega^i(T)\,\mathrm{d}\omega.$$

The writing of the quantity a is omitted.

All the absorption coefficients are defined in terms of spectral absorption coefficients. On the other hand, these may be found in terms of the functions ε, α_0, α, A_j which are directly known from experiment

$$\frac{i!}{m_i}\prod_{k=1}^{i}\alpha_k = (-1)^{i+1}(\partial^i\varepsilon/\partial x^i)_{x=0};$$

$$\frac{i!}{m_i}\prod_{k=1}^{i}\alpha_k^* = (-1)^{i+1}(\partial^i a_0/\partial x^i)_{x=0}$$

$$\frac{i!}{m_i}\prod_{k=1}^{i}\alpha_{kj} = (-1)^{i+1}(\partial^i A_j/\partial x^i)_{x=0}.$$

There is a relationship between the coefficients for a band and those for a group of bands

$$\prod_{k=1}^{i}\alpha_k = \frac{\pi}{\sigma T^4}\sum_j I_{0j}\prod_{k=1}^{i}\alpha_{kj};$$

$$\prod_{k=1}^{i}\alpha_k^* = \frac{\pi}{\sigma T_0^4}\sum_j I_{0j}(T_0)\prod_{k=1}^{i}\alpha_{kj}(T). \tag{4}$$

The above relations define the complexes

$$\prod_{k=1}^{i}\alpha_k/m_i; \qquad \prod_{k=1}^{i}\alpha_k^*/m_i; \qquad \prod_{k=1}^{i}\alpha_{kj}/m_i. \tag{5}$$

The numbers m_i are prescribed, that defines the coefficients α_i, α_i^* and α_{ij}. The numbers m_i should be such that the series or its residue convolute with the aid of the known function. The coefficients α_i at any index i should be equal. While comparing with the experiment the coefficients α_i appear to be different. Therefore, the convoluted series give only the approximate formulae or the quantities ε, a_0, a, A_j.

3. PHENOMENOLOGICAL RELATIONS

Series (1) at $m_i = i!$ is composed phenomenologically if the coefficients have the following meaning: $\alpha_1 \equiv \alpha_c$ is the proper emissivity of a medium which determines a volumetric density of a proper flow. It is also the mean absorption Planck coefficients; $\alpha_2 \equiv \alpha_*$ is the absorption coefficient in the first multiplicity of the interaction between the flow and substance; the initial flow has a spectrum of the elementary volume; α_3 is the coefficient for re-radiation of energy absorbed with the interaction of the first multiplicity, α_4 is the absorption coefficient for the earlier re-radiated flow etc. All α_i are determined, respectively.

Calculate a cone, x_0 in height, formed by an element of a space angle. Proper radiation of the medium, before passing beyond the cone boundaries, undergoes multiple interaction. The sum of dimensionless fluxes at the boundary gives the emissivity. The level of primary radiation corresponds to α_1. With no self-absorption, the flow at the boundary is equal to $\alpha_1 x_0$. At a depth x the flow is $\alpha_1 x$ and its absorption in the layer dx equals $\alpha_1 x \alpha_2\,\mathrm{d}x$. For the cone boundary primary absorption is determined by an integral $\int \alpha_1\alpha_2 x\,\mathrm{d}x = \alpha_1\alpha_2 x_0^2/2$. At a depth x the integral is equal to $\alpha_1\alpha_2 x^2/2$. Here, in the layer dx the absorbed power per fraction determined by a multiplier $\alpha_3\,\mathrm{d}x$ is re-radiated. At the cone boundary the re-radiated flow is $\alpha_1\alpha_2\alpha_3 x_0^3/6$. The further considerations give all terms of series (1) when the signs of absorbed and re-radiated flows are different. For the series of other quantities a_0, a, A_j the phenomenology is of the series at $m_i \neq i!$ reduces to the previous one if the appropriate coefficients α_i are increased by a factor of m_i/im_{i-1} ($m_i = \prod_{k=2}^{i} m_k/m_{k-1}$).

4. COEFFICIENTS IN BAND SPECTRUM

The use of discrete values of the Planck functions allows the reconstruction of a band so that the function $\alpha_\omega(\omega)$ be simple and smooth. In such a case the set of band contours is described by the formula

$$\alpha_\omega = \alpha_0 f(y). \tag{6}$$

The function $f(y)$ is very difficult to be found. Nevertheless formula (6) is used in engineering calculations. It appeared to be also useful for our analysis since the existence of the function $f(y)$ is obvious, and its most general properties are prescribed. It is assumed that the form of this function does not depend upon a pressure and temperature within rather narrow ranges of the quantities. At $S_j = \alpha_0 \Delta\omega_j$ the normalization $\int_{y'} f(y)\,\mathrm{d}y = 1$ is obtained. It is advisable to estimate α_0 so that the parameter $\Delta\omega_j$ would be close to a half-width of a non-reconstructed band. The contour is characterized by the sequence of the numbers $C_i = \int_{y'} f^i(y)\,\mathrm{d}y$ determining the sequence of

$$\frac{i!}{m_i} \prod_{k=1}^{i} \alpha_{kj} = \alpha_0^i \Delta\omega_j C_i.$$

The convolution of series (3) is highly simplified if the numbers m_i are prescribed by the equality $m_i = i!/C_i$.

As far as i increases, the numbers C_i monotonically decrease from unity to zero if a non-real rectangular contour is explicitly eliminated, that corresponds to a monotonic choice of spectral components with the greatest values of α_ω as the multiplicity of the interaction increases. Table 1 gives the numbers C_i and the mean absorption formulae corresponding to series (3) at $m_i = i!/C_i$ for the simplest contours. Thus, the coefficients m_i and formulae A_j are related to band contours. Equality (4) allows the relationship between the coefficients α_i and α_i^* and band contours to be established.

The experimental values of the functions A_j are sometimes described by the semi-empirical formulae without mentioning the type of a contour. For example, the formula [1, 2] is widely used

$$A_j = \Delta\omega_j \ln\left(1 + uf\,\frac{u+2}{u+2f}\right).$$

Here, the quantity f takes into account the pressure influence. Designating

$$uf\,\frac{u+2}{u+2f} \equiv Z$$

try to find the function $f(y)$ from the equation

$$\int_0^\infty [1 - \exp(-uf(y)]\,\mathrm{d}y = \ln(1 + Z).$$

The transformation leads to the result

$$1 - \exp(-uf(y)) = \exp(-y) \times [0{\cdot}5772 + \ln(yZ) + E_1(yZ)].$$

The function $f(y)$ found from this equation greatly depends on the optical thickness u, and it is therefore not real. There is no contour, and only at $u \to 0$ $f(y) = y\exp(-y)$ is obtained. The existence of an approximate contour would make the formula for integral absorption more convincing. The functions $f(y)$ may be obtained from the observations made at a rather high pressure.

5. APPROXIMATE FORMULAE

The series presented allow construction of a set of approximate formulae concentrating in a

Table 1. Characteristics of simplest reconstructed spectrum bands

N	$f(y)$	C_i	$\bar{A}_j$
1.	1	1	$1 - \exp(-u)$
2.	$1 - y/2$	$2/(i + 1)$	$2\{1 - [1 - \exp(-u)]/u\}$
3.	$1 - \frac{2}{3}y + \frac{1}{9}y^2$	$3/(2i + 1)$	$3[1 - 0{\cdot}5\sqrt{(\pi/u)}\,\mathrm{erf}(\sqrt{u})]$
4.	$\frac{2}{\pi}\Big/(1 + y^2)$	$\left(\frac{2}{\pi}\right)^{i-1} \prod_{k=2}^{i} \frac{2k-3}{2k-2}$	$\pi L(u/\pi)$
5.	$\exp(-y)$	$1/i$	$0{\cdot}5772 + \ln u + E_1(u)$
6.	$y\exp(-y)$	$i!/i^{i+1}$	—
7.	$2y\exp(-y^2)$	$2^{i-1}\,\Gamma\left(\frac{i+1}{2}\right)\Big/ i^{(i+1)/2}$	—

set. The formulae are composed, taking into account separately the first terms of the series and convoluting the residue with the help of some function. In this case the higher order coefficients α are equal to each other. The set and the form of the residue formula are determined by the sequence of the numbers m_i. As an example consider the emissivity. The writing of the approximate quantities a_0, a, A_j is similar.

At $m_i = i!$ the residues of series (1) convolute with the help of the exponential function. The set of the formulae is of the form:

$$\varepsilon_1 = \frac{\alpha_1}{\alpha_2} A_1(u_2); \qquad A_1 = 1 - \exp(-u_2)$$

$$\varepsilon_2 = u_1\left[1 - \frac{\alpha_2}{\alpha_3} A_2(u_3)\right]; \quad A_2 = 1 - A_1(u_3)/u_3$$

$$\varepsilon_3 = u_1\left\{1 - \frac{u_2}{2}\left[1 - \frac{\alpha_3}{\alpha_4} A_3(u_4)\right]\right\};$$

$$A_3 = 1 - 2A_2(u_4)/u_4; \qquad u_1 = \alpha_1 x;$$

$$u_2 = \alpha_2 x; \qquad u_3 = \alpha_3 x, \ldots \tag{7}$$

At $m_i = i$ the residues of the series convolute with the help of a logarithmic function

$$\varepsilon_1^* = \frac{\alpha_1}{\alpha_2} A_1^*(u_2); \qquad A_1^* = \ln(1 + u_2)$$

$$\varepsilon_2^* = u_1\left[1 - \frac{\alpha_2}{\alpha_3^*} A_2^*(u_3^*)\right]; \quad A_2^* = 1 - A_1^*(u_3^*)/u_3^*$$

$$\varepsilon_3^* = u_1\left\{1 - \frac{u_2^2}{2}\left[1 - \frac{\alpha_3^*}{\alpha_4^*} A_3^*(u_4^*)\right]\right\};$$

$$A_3^* = 1 - 2A_2(u_4^*)/u_4^*. \tag{8}$$

The quantities α^* and u^* differ from α and u by the numerical factors which to a first approximation are determined by comparing the series.

At $m_i = i!i$ the residues of the series convolute with the aid of the formula

$$A_1^0 = 0{\cdot}5772 + \ln u_2 + E_1(u_2).$$

The following set of the approximate formulae are obtained

$$\varepsilon_1^0 = \frac{\alpha_1}{\alpha_2^0} \cdot A_1^0(u_2^0)$$

$$\varepsilon_2^0 = u_1\left[1 - \frac{\alpha_2^0}{\alpha_3^0} A_2^0(u_3^0)\right]$$

$$A_2^0 = 1 - A_1^0(u_3^0)/u_3^0$$

$$\varepsilon_3^0 = u_1\left\{1 - \frac{u_2^0}{4}\left[1 - \frac{\alpha_3^0}{\alpha_4^0} A_3^0(u_4^0)\right]\right\}$$

$$A_3^0 = 1 - 4A_2^0(u_4^0)/u_4^0. \tag{9}$$

Similarly, it is possible to construct sets of other series of the formulae. For example

$$m_i = (i + 1)!, \quad i \geqslant 1$$

$$\varepsilon_1 = \frac{\alpha_1}{\alpha_2}\{1 - [1 - \exp(-u_2)]/u_2\}$$

$$m_i = i!\,(2i + 1), \quad i \geqslant 1$$

$$\varepsilon_1 = \frac{\alpha_1}{\alpha_2}\{1 - \sqrt{(\pi)}[\mathrm{erf}(\sqrt{u_2})]/2\sqrt{u_2}\}$$

$$m_i = i!\,(2i + 1), \quad m_1 = 1$$

$$\varepsilon_1 = \frac{\alpha_1}{\alpha_2}\{1 + \tfrac{2}{3}u_2 - [\sqrt{(\pi)}\,\mathrm{erf}(\sqrt{u_2})]/2\sqrt{u_2}\}$$

$$m_i = (i - 1)!\,(2i - 1)$$

$$\varepsilon_1 = \frac{\alpha_1}{2\alpha_2}\sqrt{(\pi u_2)}\,\mathrm{erf}(\sqrt{u_2}).$$

The most appropriate set is chosen by comparing the predicted values and experimental data. As far as the formula of a set is complicated, the limit thickness, at which this formula is valid, increases.

6. NUMERICAL CALCULATIONS

The approximate formulae are verified in the case of emissivity. The formulae are applicable at sufficiently small thicknesses, that is established by comparing them with the emissivity nomograms. The parts of the nomograms for small thicknesses are distorted by extrapolation. The values of those distortions could be estimated for water vapour, therefore, it is chosen as an illustration. Table 2 gives the first four coefficients

Table 2. First four coefficients of series of formulae: ε_i (first lines), ε_i^* (second lines), ε_i^0 (third lines). Emissivities are presented in the next table

t (C)	α_i (m. atm)$^{-1}$	α_2	α_3	α_4
600	3·74	33	38	38·2
	3·70	36	36	—
	3·74	70	75	75·11
800	2·74	28	36	36·25
	2·74	33	34	34·02
	2·74	60	64	64·18
1000	2·14	25	35·3	35·7
	2·14	28	29	—
	2·14	50	56	56·2
1200	1·58	22	34·2	35
	1·58	22	23·5	—
	1·58	44	56	56·63
1400	1·2	19	32·5	33·5
	1·2	19	22	22·15
	1·2	38	52	52·8
1600	0·94	16·2	30·4	31·8
	0·94	16·2	18	—
	0·94	32·4	46	46·9
1800	0·75	15	29	30·2
	0·75	15	18	—
	0·75	30	47	48·2
2000	0·62	14	28	29·2
	0·62	14	18	
	0·62	28	44	45·2

in the formulae of sets ε_i, ε_i^* and ε_i^0. Table 3 comprises the comparison of the results.

As is seen, the formula for ε_3^0 gives better approximation if higher order formulae are omitted. Table 4 shows how the range of validity of the formulae increases as far as these are complicated. The results are typical for all sets of the formulae.

7. COEFFICIENTS α_c AND α_*

The present section deals with the coefficients at atmospheric pressure which are not modified and depend only upon a temperature. The calculations discover typical difficulties in determining the coefficients α_i and help in estimating the reliability of the available experimental data. The modifications of α_c and α_* are considered in [5]. According to the first way of determining the coefficients

$$\alpha_c = (\partial\varepsilon/\partial x)_{x=0} \quad \alpha_c\alpha_* = -(\partial^2\varepsilon/\partial x^2)_{x=0}.$$

The second way results from spectroscopic formula (2). With account for the relation

$$S_j = 10^5\mu_0 S_{0j}/8{\cdot}3T$$

the formulae assume the form

$$\alpha_c = 7{\cdot}94\frac{\mu_0}{(T/1000)^5}\sum_j \frac{S_{0j}}{\lambda_j^3\left[\exp\left(\frac{\gamma}{\lambda_j T}10^4\right)-1\right]} \tag{10}$$

$$\alpha_c\alpha_* \approx 394{\cdot}2\frac{\mu_0^2}{\sqrt{(T)}(T/1000)^6} \times \sum_j \frac{S_{0j}^2}{\Delta\omega_{0j}\lambda_j^3[\exp(\gamma/\lambda_j T\,10^{-4})-1]}. \tag{11}$$

Here α_c and α_* are measured in m.atm^{-1} as μ_0 is expressed in kg/kmole, T in °K, S_{0j} in cm^{-1}/gm^{-2}, λ in μ. Formula (11) is approximate since the numerical factor is taken to be the same for all bands. It depends upon a rotational structure. The numerical factor is chosen so that at t = 800°C $\alpha_* = 50{\cdot}6$ (m. atm)$^{-1}$ as in [4] where the

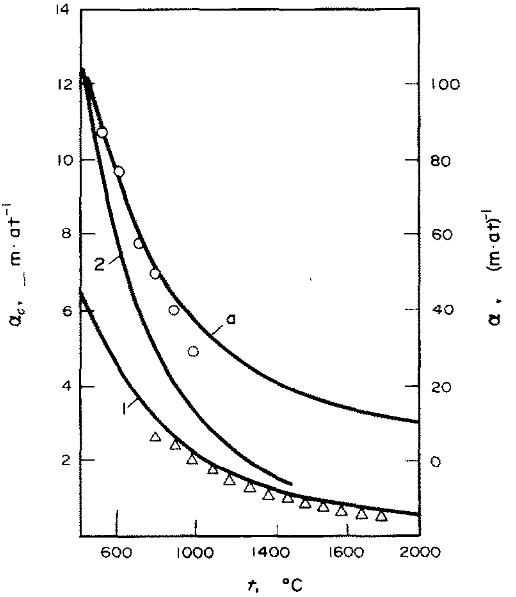

FIG. 1. Radiation and absorption coefficients of water vapour. 1—coefficients α_c calculated by formula (10) with band intensities from [6]. Points △ are calculated by formula of [3]. 2—curve taken from [11] which was obtained by formula different from (10). Curve a shows function $\alpha_*(T)$ calculated by formula (11). Points ○ are plotted according to [4].

Table 3. Water vapour emissivities. First lines: at x $\leqslant$ 0·03 *m.atm* [3], *at* x > 0·03 *m.atm* [4]. *Second lines*—ε_3 (7), *third lines*—ε_2^* (8), *fourth lines*—ε_3^0(9). *Coefficients of the formulae are given in Table 2. Figures after a decimal point are presented*

x (m. atm) \ t (°C)	600	800	1000	1200	1400	1600	1800	2000
0·002	00723	00535	00419	00305	00236	00184	00148	00122
	00723	00533	00417	00309	00236	00185	00148	00122
	00714	00531	00416	00309	00236	00185	00148	00122
0·01	0317	0238	0189	0142	0110	00869	00704	00576
	0319	0240	0190	0142	0110	00871	00699	00580
	0316	0237	0189	0143	0110	00872	00700	00581
	0317	0239	0190	0143	0110	00871	00699	00580
0·03	0732	0568	0463	0359	0284	0230	0188	0156
	0729	0574	0468	0360	0284	0230	0186	0156
	0753	0574	0468	0366	0288	0231	0187	0157
	0737	0575	0469	0361	0285	0230	0187	0156
0·06	107	089	0742	0604	0500	0410	0350	0280
	106	089	0750	0595	0482	0400	0326	0275
	118	092	0762	0614	0495	0401	0329	0279
	112	090	0750	0600	0484	0398	0328	0275
0·2	204	178	154	131	112	0962	0826	0720
		175	161	134	114	0974	0813	0696
	216	177	151	130	112	0905	0780	0684
	201	170	146	130	110	0930	0802	0676
0·5	300	270	240	210	185	162	142	126
					204	172	152	133
	303	266	232	211	199	155	142	131
	301	264	230	219	194	166	148	125
1	380	340	308	275	244	217	191	170
							181	166
	371	359	316	275	311	232	227	217
	389	340	305	271	250	217	197	167

Table 4. Water vapour emissivities at 800°C according to formulae of a set at $m_i = i$. *Coefficients of formulae are in* $(m.atm)^{-1}$. $\alpha_1 = 2{\cdot}74$ *and* $\alpha_2 = 33$, $\alpha_3 = 34$, $\alpha_4 = 34{\cdot}02$. *Values of* ε *according to the nomogram in* [4]. *Figures after a decimal point are presented*

x, (*m. atm*)	ε_1^*	ε_2^*	ε_3^*	ε
0·002	00531	00531	00530	—
0·01	0237	0237	0237	0240
0·03	0571	0574	0574	0560
0·06	0907	0918	0918	089
0·1	121	124	124	122
0·2		177	176	178
0·5		266	261	270
1			340	340

Table 5. Absorption coefficients $\alpha_* \equiv \alpha_1$ *for carbon dioxide gas taken from various sources. Values of* α'_* *are calculated by Shack's formula* [13]

t (°C)	α_* (m.atm)$^{-1}$	
	[4]	α'_*
400	524	620
500	502	—
600	452	500
700	403	—
800	366	430
900	327	—
1000	287	370
1100	252	—
1200	232	330
1400	220	300

calculation is made by the first way over a finite temperature range, and the values of α_* are presented implicitly. Thus, formula (11) is used only for determining the function $\alpha_*(T)$.

Figure 1 gives the data on α_c and α_* for water vapour taken from different literature sources. Curve 1 and points with a triangular contour show the agreement of our calculations of α_c with the aid of both ways. According to the second way the spectroscopic data are taken from [6] without any change. For a 2·7μ-band the value of S_{0j} slightly differs from 23·8 cm^{-1}/gm^{-2} in [7], 24·9 in [8] and 25·2 in [9]. The discrepancies in the data on 6·3μ band are more considerable, and in [10] the temperature dependence S_{0j} is taken although the band is fundamental. Following Curve 2 in Fig. 1 [11] the intensities of bands are taken more high than here. The shape of the curve for α_* (Curve a) satisfactorily coincides with the data of [4] if at $t = 1000°C$ the point is taken into consideration. Figure 2 compares quantities α_c for carbon dioxide gas.

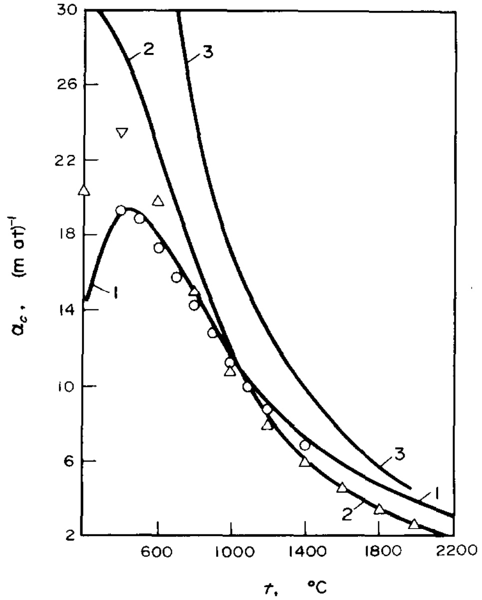

FIG. 2. Carbon dioxide gas radiation coefficients. 1—calculation by Shack's formula [13]. 2—calculation by formula (10) with band intensities from [6]. ○—points taken from [4]. △—points calculated by formula (10) with changing band intensities.

The results obtained by both ways greatly disagree. In [11] the disagreement is explained by a part of the nomogram at small thicknesses. Apparently, the explanation is insufficient. Spectroscopic data, according to which Curve 3 is constructed [11] are overestimated. A 4·3μ band plays an essential role, for which the intensity of 151·5 cm^{-1}/gm^{-2} is mentioned in [12] and elsewhere. In [6] Edwards *et al.* use the value of 110 cm^{-1}/gm^{-2}. During a special fitting procedure of the spectroscopic data satisfying Curve 1 they appeared to be beyond the limits of errors. Furthermore, assuming that the averaging of the Planck function in a 4·3μ band leads to a large error, the band was divided into two parts. In such a way the points with a triangle contour (Fig. 2) are obtained. However, there is still no agreement. These attempts have shown that the lower part of the nomogram $\varepsilon(x, T)$ for CO_2 is distorted. The values of α_* obtained by the first way are given in Table 5.

The accuracy of the coefficients α_i completely depends upon that of spectroscopic data and nomograms for $\varepsilon(x, T)$ at small thicknesses. For a carbon dioxide gas the spectroscopic data are contradictive, and the distortions of the nomogram are insufficiently revealed. For water vapour the spectroscopic data are more reliable, and the underestimate of the nomogram at $x \leqslant 0·03$ may be found.

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COEFFICIENTS D'ABSORPTION SPECTRALE ET INTEGRALE D'UN MILIEU

Résumé—L'absorption d'un milieu dans une bande spectrale, dans un spectre total et l'émissivité sont décrites par des séries convergentes. On a étudié les coefficients des séries. Les systèmes de formules approchées sont construits par convolution des résidus des séries. Ces formules sont vérifiées pour l'émissivité de la vapeur d'eau. Deux coefficients d'ordre plus faible connus dans la documentation sont discutés comme un cas particulier.

ABSORPTIONSKOEFFIZIENTEN FÜR DAS INTEGRALE SPEKTRUM EINES MEDIUMS

Zusammenfassung—Die Absorption und die Emission eines Mediums wird sowohl für einen spektralen Bereich, als auch für das Gesamtspektrum durch konvergente Reihen beschrieben. Die Reihenkoeffizienten wurden untersucht. Die Serie der approximierten Formeln wurde durch Faltung der Residuen der Reihen aufgebaut. Diese Formeln wurden für die Emission von Wasserdampf verifiziert. Zwei aus der Literatur bekannte Koeffizienten niedriger Ordnung wurden als Sonderfall diskutiert.

ИНТЕГРАЛЬНЫЕ ПО СПЕКТРУ КОЭФФИЦИЕНТЫ ПОГЛОЩЕНИЯ СРЕДЫ

Аннотация—Поглощение среды в полосе спектра, в полном спектре и степень черноты описываются сходящимися рядами. Проведено исследование коэффициентов рядов. Путем свертывания остатков рядов составлены серии приближенных формул. Они проверены на примере степени черноты водяного пара. Два коэффициента низших порядков, известные в литературе, обсуждаются отдельно.